Specialization in a Dynamic Trade Model: An Overlapping Generations Case

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Discussion Paper No.04-07
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October 13, 2004

Abstract

We explore a small-open economy with overlapping generations to understand what is the determinant of trade pattern in the dynamic trade theory. In the representative agent model (Baxter (1992) ), even if there are two goods and two production factors, opening up to trade leads to perfect specialization. Using the overlapping generations model, we find that the heterogeneity of the economic agents makes imperfect specialization occur generally. We also find that whether the stability condition holds or not is crucial for the determination of the production pattern.

Keywords: Specialization patterns; Dynamic Heckscher-Ohlin model; Overlapping generations.

JEL Classification Numbers: F41, O41
1 Introduction

We explore a small-open economy with overlapping generations to understand what is the determinant of trade pattern in the dynamic trade theory. In the representative agent model (Baxter (1992) and Ono and Shibata (1994)), even if there are two goods and two production factors, opening up to trade leads to perfect specialization. This contrasts with the static trade theory, in which the slight difference in technology causes the slight difference in factor prices only but does not specialization. In the small-open economy context, this means perfect specialization generally occurs in the dynamic trade theory and does not in the static trade theory.\(^1\)

Using the overlapping generations model (OLG model), which is constructed in Weil (1989) we find that the inflow of a new economic agent has a crucial role in determining the trade pattern.\(^2\) In this model, new agent enters the economy with zero asset and financially disconnected from other generations. Thus, there is heterogeneity among households.

In the representative agent model, heterogeneity is assumed away and the path of asset accumulation is unique. Suppose that the two sectors operate, the change in the factor price must obey the Stolper-Samuelson theorem as the world price changes. The interest rate determined by the world price contradicts with the optimal asset accumulation as long as the world price and the autarky price differs from each other. On the other hand, in the OLG model, the social path of asset accumulation is dependent on the behavior of the new agent. Thus, the interest rate and the level of total capital can change according to the world price and imperfect specialization can occur.

Recently Bianconi (1995), and Cremers (2001) also show that an economy may well produce both commodities in the long run using the OLG model. In these papers, they use a two-period OLG model while we use the continuous-time OLG model. Our approach makes it clear that the difference between the determinants of the specialization pattern in the representative agent model and OLG model is not the finite time horizon of agents but the disconnect-ness of agents and confirms the robustness of the result obtained in Bianconi (1995), and Cremers (2001).

In addition, we analyse the effect of the world price on the production pattern. In general, the production pattern is determined by the world price and the total capital-labor ratio when there are two factors of production and two sectors with constant returns to scale. Since the factor of production is endogenously supplied in our model, our analysis is rather different from that in the standard static trade theory. We must consider the path of the capital accumulation. We find that whether the stability condition holds or not is crucial for the determination of the production pattern.\(^3\)

Some papers analyse the OLG model in an open economy. Buiter (1981) extends Blanchard-

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\(^1\)Kaneko (2000) shows that this property holds even in the endogenous growth model.

\(^2\)The other standard continuous time version of an OLG model is presented by Blanchard (1985). In his model, at every instance a new generation born and every generation faces the same probability of death.

\(^3\)Oniki and Uzawa (1965) and Stiglitz (1970) analyzed the long-run trade pattern of the dynamic trade model in the two-country model. They focused on the difference of the saving behavior between the countries.
type model to an open economy and examine the effect of the fiscal policies. In his model, the production pattern and therefore the trade pattern are exogenous. Matsuyama (1988) discusses the possibility of a Laursen-Metzler Effect, that is, positive impact of the terms of trade on a current account, in a small-open economy populated with two-period overlapping generations. He points out that it depends on the factor intensity of the export sector. In his model, the economy opens financially and the determinant of the production pattern is different from ours.

The paper is organized as follows. Section 2 presents the model. We discuss the determinant of specialization pattern in section 3. The production pattern is analyzed in section 4. Section 5 concludes the paper.

2 The Model

2.1 The Household

The demographic structure of the model follows Weil (1989) which points out the crucial role of the arrival of new agents and obtains the feature of overlapping generations model. The economy consists of many infinitely-lived agents. In an interval of time of length $dt$, $dN(t) = nN(t) dt$, $n \geq 0$, new and identical infinitely-lived agents appear in the economy with zero asset, so that the total number of agents alive at time $t$ is $N(t) = N(0) \exp(nt)$. When $n = 0$, the economy consists only of the original infinitely-lived agents, which means that the model is reduced to the representative agent one.

A representative household born at time $s$ (which we call s household) maximizes his lifetime utility subject to the instantaneous budget constraint:

$$\max \int_{s}^{\infty} [\alpha \ln c_{1}(s,t) + (1-\alpha) \ln c_{2}(s,t)] \exp[-\rho(t-s)] dt$$

s.t. $\dot{a}(s,t) = r(t)a(s,t) + w(t) - c_{1}(s,t) - p(t)c_{2}(s,t).$ \hspace{1cm} (2)

A dot denotes a time derivative. $a(s,t)$ and $c_{i}(s,t)$ denote, respectively, nonhuman wealth and consumption of commodity $i$ at period $t$ of representative household who is born at $s$. $p$ represents the price of commodity 2 in terms of commodity 1. $r(t)$ is the riskless interest rate on assets. The first order conditions give

$$c_{1}(s,t) = [r(t) - \rho]c_{1}(s,t),$$

$$p = \frac{1 - \alpha}{\alpha} \cdot \frac{c_{1}^{t}(s,t)}{c_{2}^{t}(s,t)}.$$ \hspace{1cm} (4)

The transversality condition is required.

$$\lim_{t \to \infty} \lambda a(s,t) \exp \left( - \int_{s}^{t} r(v) dv \right) = 0,$$ \hspace{1cm} (5)

3
\(\lambda\) is a costate variable of Hamiltonian. Under the transversality condition, equation (3) gives the consumption function of the commodity 1:

\[
c_1(s, t) = \alpha \rho \left[ h(t) + a(s, t) \right],
\]

where \(h(t) = \int_0^\infty w(u) \exp\left( - \int_u^t r(v) dv \right) du\) denotes human wealth at \(t\) and is, by assumption, age independent.

For any variable \(x(s, t)\) pertaining to an individual agent, define the corresponding aggregate magnitude \(x(t)\) par capita as

\[
x(t) = \frac{N(0)x(0, t) + \int_0^t x(s, t) dN(s)}{N(t)}.
\]

Using this definition, from (6) it follows that per capita aggregate consumption is a constant fraction of total wealth (For ease of exposition, we suppress the time index of an aggregate variable per capita in the following.).

\[
c_1 = \alpha \rho \left[ h + a \right].
\]

and (4) can be transformed into

\[
p = 1 - \alpha \frac{c_1}{c_2}.
\]

The law of motion of aggregate consumption of commodity 1 is

\[
\dot{c}_1/c_1 = r - \rho - \alpha p a/c_1.
\]

The element in (10), \(apna/c_1\), captures the effect of the entry of the new people with zero assets. Since \(\alpha\) is the propensity to consume commodity 1 out of wealth, \(\alpha p a\) is per capita aggregate consumption associated with \(a\). Thus, the entry of the new people lowers the per capita aggregate consumption by the amount of \(apna\). Finally, the division by \(c_1\) gives the contribution of these term to the reduction in the growth rate of the aggregate per capita consumption.

From the flow budget constraint, aggregate nonhuman wealth evolves according to

\[
\dot{a} = ra - c_1 - p c_2 - w - n a.
\]

### 2.2 The Optimal Conditions of Firms

There are two production sectors and two factors of production; physical capital and labor. The first sector of the economy produces commodity 1 (capital good) that is used for both investment and consumption. The second sector produces commodity 2 (a consumption good) used only for consumption. We assume that the production functions satisfy constant returns to scale:

\[
y_1 = \gamma_1 f_1(k_1),
\]

\[
y_2 = \gamma_2 f_2(k_2),
\]
where \( y_i \) is per capita aggregate production of commodity \( i \), \( k_i \) the physical capital to labor ratio employed in the \( i \)th sector, \( \gamma_i \) the ratio of the employed labor in sector \( i \) to the total labor. We assume that \( f_i (k_i) \) satisfy the standard neoclassical assumptions and Inada conditions.

\[
\frac{df_i}{dk_i} > 0, \quad \frac{d^2f_i}{dk_i^2} < 0, \\
\lim_{k_i \to 0} \frac{df_i}{dk_i} = \infty, \quad \lim_{k_i \to \infty} \frac{df_i}{dk_i} = 0, \quad (i = 1, 2)
\]

where \( f_i' = \frac{df_i}{dk_i} \), \( f_i'' = \frac{d^2f_i}{dk_i^2} \), \( (i = 1, 2) \). The profit maximization by competitive firms equates the value of marginal product of each factor input with the interest rate and the wage rate. Thus as long as the two sector operates we have:

\[
f_1' (k_1) = pf_2' (k_2) = r. \tag{14}
\]

\[
f_1 (k_1) - k_1 f_1' (k_1) = p \left[ f_2 (k_2) - k_2 f_2' (k_2) \right] = w. \tag{15}
\]

From (14) and (15), we derive that every variable depends only on \( p \).

\[
k_1 = k_1 (p), \quad k_2 = k_2 (p), \quad r = r (p), \quad w = w (p). \tag{16}
\]

Namely, the interest rate and the wage rate depends on the relative price as follows, which implies the Stolper-Samuelson theorem.

\[
r' (p) = \frac{f_2 (k_2)}{k_2 - k_1}, \quad w' (p) = \frac{f_2 (k_2) k_1}{k_1 - k_2}.
\]

2.3 The Market Equilibrium Conditions

We assume that the physical capital and labor are fully employed, so that

\[
\gamma_1 k_1 (p) + \gamma_2 k_2 (p) = k, \tag{17}
\]

\[
\gamma_1 + \gamma_2 = 1, \tag{18}
\]

where \( k \) is aggregate capital stock per capita.

From the above two equations, the fraction of labor devoted to each sector can be expressed as

\[
\gamma_1 = \gamma_1 (k, p), \quad \gamma_2 = \gamma_2 (k, p). \tag{19}
\]

The equilibrium conditions in the markets of the two commodities and capital are

\[
y_1 = c_1 + k + nk, \tag{20}
\]

\[
\gamma_2 (k, p) f_2 (k_2 (p)) = c_2, \tag{21}
\]

\[
k = a. \tag{22}
\]
2.4 Characterization of the Dynamic System

From (9) and (21), we know that the relative price can be represented by the function of \( k \) and \( c_1 \):

\[
p = p(c_1, k).
\]

(23)

The law of motion of physical capital and consumption of commodity 1 are as follows from (10) and (11):

\[
\dot{c}_1 = r(p(c_1, k)) - \rho - c_1 - \alpha \rho nk. \tag{24}
\]

\[
k = r(p(c_1, k)) + w(p(c_1, k)) - c_1/\alpha - nk. \tag{25}
\]

Equations (24) and (25) constitute the autonomous dynamic system.

3 A Small-Open Economy

3.1 The Possibility of Specialization

When the closed economy opens trade as a small-open economy, the relative price is considered as exogenous. In the following argument we assume that a financial asset on installed physical capital is not traded internationally. The profit maximization conditions for firms are the same as (14) and (15). Thus, given the world price, equations in (16) determine \( r \) and \( w \). Since we do not allow the foreign asset accumulation, the motion of the level of \( c_1 \) and \( k \) are expressed as (24) and (25). Thus, the steady state equilibrium conditions can be written as

\[
[r(p^w) - \rho] c_1 - \alpha \rho nk = 0, \tag{26}
\]

\[
r(p^w) k + w(p^w) - c_1/\alpha - nk = 0. \tag{27}
\]

If the equilibrium is saddle-stable, equations (26) and (27) solve the new steady state value of \( k \) and \( c_1 \). From (19), we can derive \( \gamma_1 \) and \( \gamma_2 \). When these variables are greater than 0 and less than 1, it implies that perfect specialization does not occur in the steady state. When we consider the case of the representative agent \( (n = 0) \), the steady state equilibrium conditions are reduced to

\[
r(p^w) - \rho = 0, \tag{28}
\]

\[
r(p^w) k + w(p^w) - c_1/\alpha = 0. \tag{29}
\]

(28) implies that the steady state equilibrium at which the two sectors operate (i.e. imperfect specialization occurs) does not realize as long as the level of the world price happens to make the interest rate equal to the subjective discount rate (See Baxter (1992) and Ono and Shibata (1994) in detail). The difference between Baxter (1992) (or Ono and Shibata (1994)) and this paper stems from the inflow of new agent \( (n > 0) \).
The intuition behind this result is as follows. In the representative agent model, heterogeneity is assumed away and the path of asset accumulation is unique. When the two sectors operate, the factor price changes according to the Stolper-Samuelson theorem. Thus, the interest rate determined by the world price according to the Stolper-Samuelson theorem contradicts with the optimal asset accumulation as long as the world price and the autarky price differs from each other. On the other hand, in the OLG model, the agent is financially disconnected from each other and the social path of asset accumulation is dependent on the behavior of the new agent. Thus, the interest rate and the level of total capital can change according to the world price and the imperfect specialization can occur.

**Lemma.** In the dynamic trade model with overlapping generations, perfect specialization does not necessarily occurs even if the world price is not equal to the autarky price.

In the next subsection, we examine the effect of the world price on the production pattern taking the market equilibrium conditions and the stability condition into account.

### 3.2 The Long-Run Production Pattern

Under our assumption that there is no world financial market, the interest rate is determined by the world price as long as the two sectors operate. As shown in the appendix, the level of the interest rate determines the stability of dynamic system. Thus, we examine the determination of production pattern in the steady state in three cases: (i) \( \rho < r(p^w) < \rho + n \), (ii) \( r(p^w) < \rho \), (iii) \( \rho + n < r(p^w) \).

(i) \( \rho < r(p^w) < \rho + n \)

When this condition holds, the system has a locally saddle stable path. In the long run, the level of aggregate capital stock per capita reaches its steady state level. As the capital accumulates, the labor force devoted to each sector changes. From (26) and (27), the steady state level of aggregate capital stock per capita is represented as below.

\[
k = \frac{\rho - r(p^w)}{r(p^w)(r(p^w) - \rho + n)} w(p^w).
\]

Thus, using (17) and (18) the labor devoted to sector 1, \( \gamma_1 \), is represented as

\[
\frac{\rho - r(p^w)}{r(p^w)(r(p^w) - \rho + n)} w(p^w) - k_2(p^w) < \frac{k_1(p^w)}{k_1(p^w) - k_2(p^w)}.
\]

For the economy to specialize imperfectly, it must be greater than 0 and less than 1 to satisfy (18). Thus, the following condition must hold.

\[
k_2(p^w) < \frac{\rho - r(p^w)}{r(p^w)(r(p^w) - (\rho + n))} w(p^w) < \frac{\rho}{k_1(p^w)} < k_1(p^w), \text{ if } k_2(p^w) < k_1(p^w)
\]
This means that the new steady state level of aggregate capital stock per capita must lie between
$k_1(p^w)$ and $k_2(p^w)$.

(i) $r(p^w) \leq \rho$

From (3), we know that the interest rate is so low that the level of the consumption of all
household lowers gradually. Since the world price is given, aggregate human wealth per capita, $h$, is constant at $w(p^w)/r(p^w)$ by the definition of $h$. Thus, from (8) we know that aggregate asset per capita, $a$, de-cumulates as the level of consumption lowers. Since $a = k$ in equilibrium, the capital stock becomes too small for the two sectors to operate and the economy would specialize
in a labor intensive good in the long run.

(ii) $\rho + n \leq r(p^w)$

From (24) and the condition $\rho + n \leq r(p^w)$

$$c_1 = [r(p^w) - \rho] c_1 - \alpha \rho n k \geq n(c_1 - \alpha k).$$

Since $h$ must be positive, from equation (8) $c_1$ is greater than $\alpha \rho a$. Since $a = k$ in equilibrium, the above equation implies that $c_1$ grows throughout. It means that the high interest rate leads individuals to postpone consumption. Thus, from (8), aggregate capital stock per capita, $k$, also
increases with $h$ constant and the economy would specialize in a capital intensive good in the long run.

Summarizing the above argument, we establish the following proposition.

**Proposition.** When the stability condition is satisfied and the new steady state level of aggregate
capital stock per capita is between $k_2(p^w)$ and $k_1(p^w)$, the imperfect specialization occurs in the
long run. Otherwise, the economy specializes in one of the two commodities perfectly.

Note that even though the perfect specialization occurs in the overlapping generations model,
the determinant of specialization is completely different from the representative agent model.

\[4\text{In this model, the production pattern and the world price is not related monotonously. This is because the level of the factor of production, } k, \text{ is endogenously determined and is not related to the world price monotonously.}\]
4 Conclusion

In this paper, we examine the determination of the production pattern in the OLG model. We show that imperfect specialization occurs generally even in the dynamic trade model. This is because the new born agent enters the economy with zero asset generates heterogeneity in the economy. Moreover, we find that the stability condition is one of the determinant of the production pattern.

We examine the production pattern and do not the trade pattern of the small-open economy. Since the factor of production is endogenously determined, even though we adopt simple production functions and a utility function, a change in excess supply caused by the world price is difficult to analyse.

Some extensions would may be useful. First, we can extend our analyses to a two-country model. In the case of the two-countries model, the terms of trade is endogenized. We can find the relationship between a structure of demand and specialization. Next, the welfare analyse would be meaningful. When the two sectors operate, a change in the terms of trade influences the factor prices (the Stolper-Samuelson effect). Since the households accumulate different level of asset, the effect of a change in the terms of trade would be different for each generation.

5 Appendix

When the economy opens to trade, the relative price of the commodities is exogenously given and the dynamic system becomes

\[\dot{c}_1 = [r(p^w) - \rho] c_1 - \alpha \rho n k,\]  
\[\dot{k} = r(p^w)k + w(p^w) - c_1 / \alpha - nk.\]  

(30)  

(31)

Linearizing (30) and (31), we obtain

\[\begin{pmatrix} \dot{c}_1 \\ \dot{k} \end{pmatrix} = \begin{pmatrix} r - \rho & -\alpha \rho n \\ -1/\alpha & r - n \end{pmatrix} \begin{pmatrix} dc_1 \\ dk \end{pmatrix}.\]

The determinant of the coefficient matrix is given by \((r(p^w) - \rho)(r(p^w) - \rho - \rho n)\). The aggregate capital stock per capita is given at any point in time and the consumption level can jump at any point in time. Thus, for the system to have a unique stable path, it must have one negative root and one positive root. When the determinant of the matrix is negative, that is, \(r(p^w) < \rho + n\), the equilibrium is locally saddle point stable.

References


